Lattice Models

LATTICE MODELS AROSE IN STATISTICAL MECHANICS AS DISCRETE SYSTEMS THAT EXHIBIT INTERESTING BEHAVIOR SUCH AS PHASE TRANSITIONS AND SCALING PHENOMENA.

SOLVABLE MODELS ARE USUALLY 2-DIMENSIONAL. HISTORICALLY THE FIRST TO BE SOLVED WAS THE 2-DIMENSIONAL ISING MODEL (ONSAGER, 1944). BUT WE WILL START WITH BAXTER'S ANALYSIS OF THE FIELD-FREE SIX-VERTEX MODEL.

ALTHOUGH THE FIRST APPLICATIONS OF LATTICE MODELS WERE TO STATISTICAL MECHANICS THEY ARE CLOSELY RELATED TO HEISENBERG SPIN CHAINS IN QM. THROUGH QUANTUM GROUPS THEY ARE CONNECTED TO KNOT THEORY AND (POTENTIALLY) TOPOLOGICAL QUANTUM COMPUTING. BELAVIN, POLYAKOV AND ZAMOLODCHIKOV SHOWED THAT SCALING PROPERTIES OF SOLVABLE MODELS AT A CRITICAL POINT ARE PREDICTED BY CONFORMAL FIELD THEORY. RECENTLY THEY HAVE APPEARED IN NEW AREAS: INTEGRABLE PROBABILITY, NONSYMMETRIC MACDONALD POLYNOMIALS AND REPRESENTATIONS OF p-ADIC GROUPS.

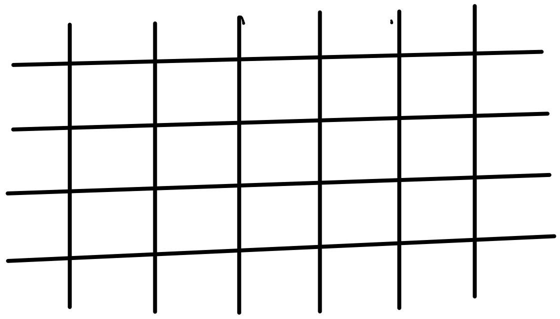
The Six Vertex Model

THE SIX VERTEX MODEL ORIGINATED IN LINUS PAULING'S 1937 PAPER ON ICE, SO THESE ARE ALSO CALLED "ICE TYPE" MODELS.

THE CORRESPONDING HEISENBERG SPIN CHAIN IS THE XXY HAMILTONIAN. ANALYSIS BY FADDEEV AND HIS STUDENTS, PARTICULARLY KULISH, SKLYANIN AND RESHETIKHIN LED TO THE INVENTION OF QUANTUM GROUPS BY DRINFELD AND (INDEPENDENTLY) JIMBO.

THE FIELD FREE SIX VERTEX MODEL WAS PROVED SOLVABLE BY LIEB AND SUTHERLAND. BAXTER INTRODUCED THE YANG-BAXTER EQUATION WHICH IS NOW UNDERSTOOD IN TERMS OF QUANTUM GROUPS.

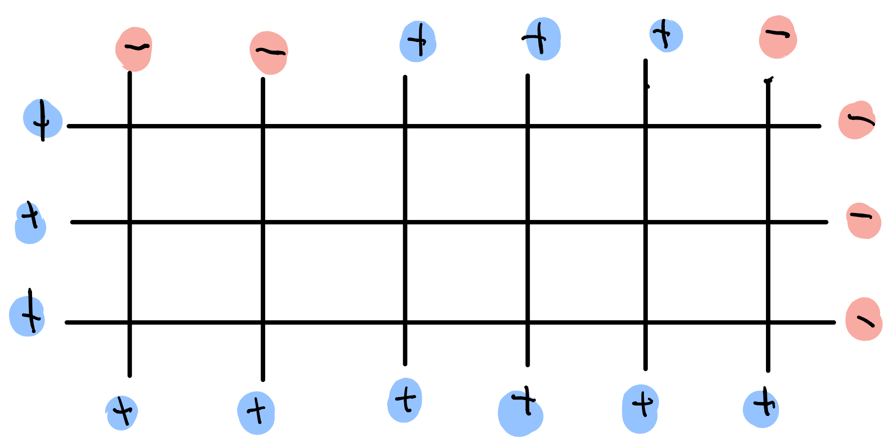
START WITH A RECTANGULAR GRID:



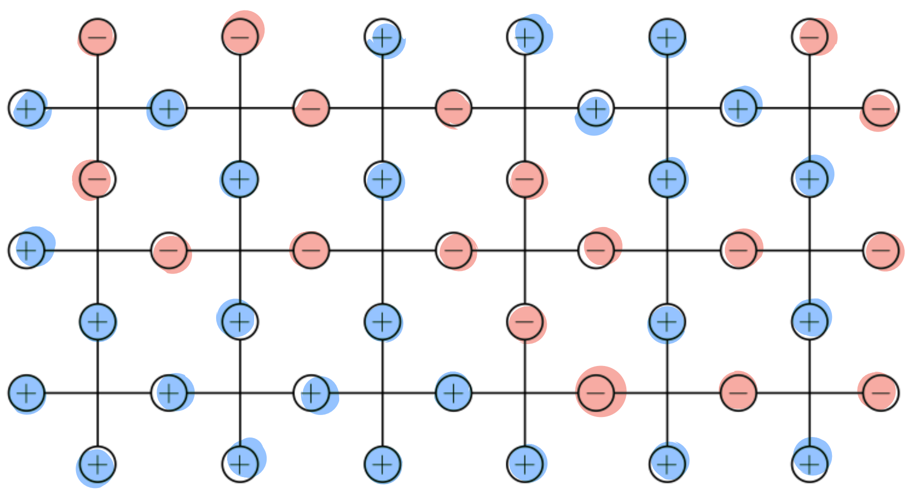
THE GRID CAN BE INFINITE, OR FINITE WRAPPED ON A TORUS, OR FINITE WITH FIXED "BOUNDARY DATA" TO BE EXPLAINED.

A STATE OF THE SYSTEM ASSIGNS A "SPIN" + OR − TO EACH EDGE.

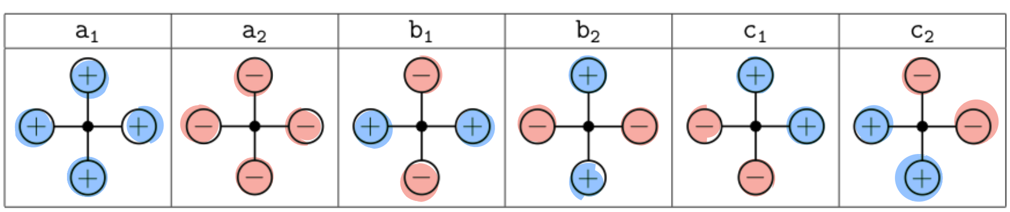
IF THERE ARE BOUNDARY EDGES THE SPINS OF THE BOUNDARY EDGES ARE FIXED. THESE ARE THE BOUNDARY CONDITIONS



A STATE OF THE SYSTEM ASSIGNS SPINS TO ALL EDGES:



AT A VERTEX ONLY SIX CONFIGURATIONS ARE ALLOWED:



✓

WE CHOOSE SIX VALUES $a_1, a_2, b_1, b_2, c_1, c_2$ FOR THESE STATES WHICH IN STATISTICAL MECHANICS MEASURE THE ENERGY OF THE CONFIGURATION. THESE ARE CALLED BOLTZMANN WEIGHTS.

THE WEIGHT $\beta(\Delta)$ OF A STATE Δ IS THE PRODUCT OF THE BOLTZMANN WEIGHTS OF THE VERTICES. THE PARTITION FUNCTION OF THE SYSTEM $\mathfrak{S}$ IS

$$Z(\mathfrak{S}) = \sum_{\text{STATES } \Delta} \beta(\Delta)$$

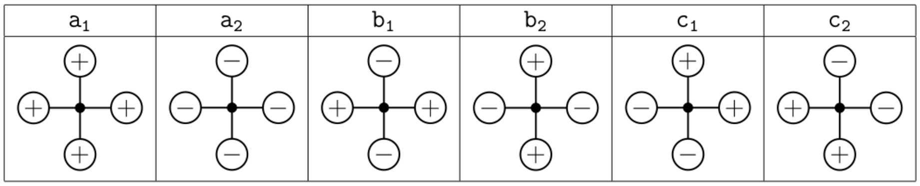
IS THE SUM OF THE WEIGHTS OF ALL STATES.

IN STATISTICAL MECHANICS THE PROBABILITY OF A STATE Δ IS:

$$\frac{\beta(\Delta)}{Z(\mathfrak{S})}$$

THUS WE HAVE A PROBABILITY DISTRIBUTION AND COULD ASK (FOR EXAMPLE) WHETHER THE SPINS AT REMOTE PARTS OF THE GRID ARE STRONGLY CORRELATED. THE SYSTEM IS THUS CAPABLE OF PHASE TRANSITIONS. DEEPER, THE PARTITION FUNCTION CONTROLS THE THERMODYNAMIC PROPERTIES OF THE SYSTEM.

SOME TYPES OF SIX VERTEX MODEL:



FIELD-FREE: $a_1 = a_2,\ b_1 = b_2,\ c_1 = c_2$

SOLVED BY LIEB, SUTHERLAND AND BAXTER.

A KEY EXAMPLE IN QUANTUM GROUP THEORY.

$$U_q(\widehat{sl}_2) \qquad \frac{1}{2}(q+q^{-1}) = \frac{a^2+b^2-c^2}{2ab} \quad \checkmark$$

"QUANTUM AFFINE sl_2"

FREE-FERMIONIC: $a_1 a_2 + b_1 b_2 = c_1 c_2$

CONTAINS THE ABOVE $U_q(\widehat{sl}_2)$ WITH $q = \sqrt{-1}$

BUT IT ALSO CONTAINS EXAMPLES RELATED

TO $U_q(\widehat{gl}(1|1))$ SUPERSYMMETRIC

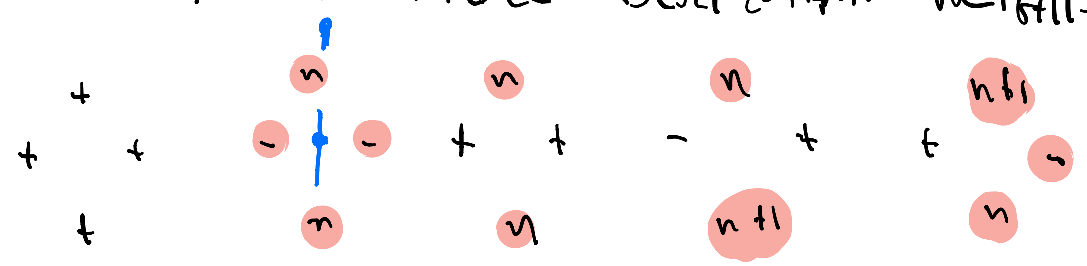
STOCHASTIC: $a_1 = a_2 = 1,\ b_1 + c_2 = 1,\ b_2 + c_1 = 1$

THESE SIX VERTEX MODELS GIVE RISE

TO MARKOV CHAINS.

Bosonic and Fermionic Models

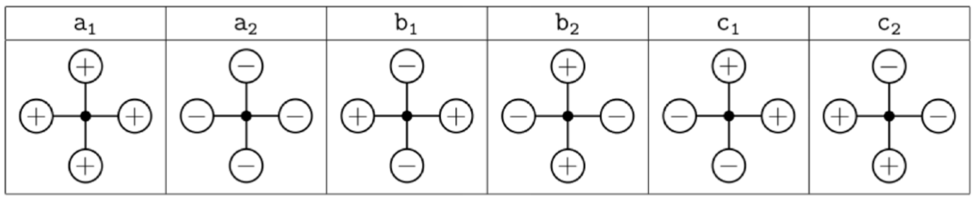
A type of model closely related to the 6 vertex model allows $-$ spins to occur with multiplicity > 1 on vertical edges. Thus if n $(n \geq 0)$ denotes a state where an edge carries n $-$ spins we have the following possible Boltzmann weights:



These systems are called **Bosonic**.

In the usual 6-vertex model we only allow $n \leq 1$ and such a system may be called **Fermionic**.

Field-Free Six Vertex Model

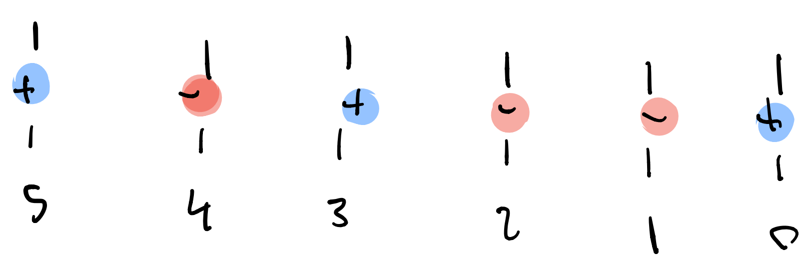


$a_1 = a_2 = a, \quad b_1 = b_2 = b, \quad c_1 = c_2 = c$

$$\Delta = \frac{a^2+b^2-c^2}{2ab} = \frac{1}{2}(q+q^{-1}).$$

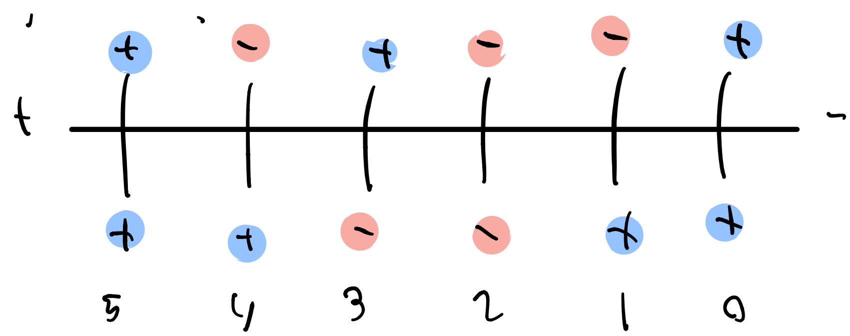
A GOOD WAY TO STUDY THE PARTITION FUNCTION INVOLVES ROW-TRANSFER MATRICES.

CONSIDER A SEQUENCE OF VERTICAL EDGES AS A VECTOR SPACE V. ASSIGNING SPINS PICKS OUT A VECTOR:



$$v_4 \wedge v_2 \wedge v_1 \in V$$

NOW A SINGLE ROW OF "ICE" GIVES



SUMMING OVER STATES (ASSIGNMENTS TO HORIZONTAL EDGES HERE) GIVES A VALUE, THE COEFFICIENT OF

$v_3 \wedge v_2$ IN $T(v_4 \wedge v_2 \wedge v_1)$.

THE MATRIX T IS THE

ROW TRANSFER MATRIX.

BOTTOM ROW TOP ROW

$$Z(\mathfrak{S}_{\substack{\text{single}\\ \text{row}\\ \text{system}}}) = \langle v_3 \wedge v_2 \,|\, T \,|\, v_4 \wedge v_2 \wedge v_1 \rangle$$

$T \in \mathrm{End}(V)$

Baxter's Insight

IN AN n-ROW SYSTEM IF $T_1, \ldots, T_n$ ARE THE ROW TRANSFER MATRICES AND IF $\xi, \eta \in V$ ENCODE THE TOP AND BOTTOM BOUNDARY CONDITIONS

$$Z(\mathfrak{S}) = \langle \eta | T_n \cdots T_1 | \xi \rangle .$$

BAXTER SAW:

(1) WE SHOULD STUDY THE PARTITION FUNCTION BY DIAGONALIZING THE ROW TRANSFER MATRICES

(2) THIS IS BEST DONE BY EMBEDDING THE MATRIX IN A LARGE COMMUTING FAMILY

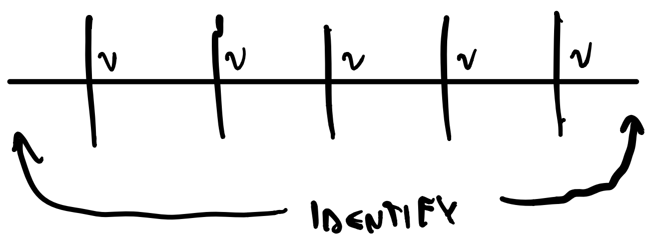
(3) THERE EXISTS A POWERFUL TOOL FOR PROVING COMMUTATIVITY OF ROW TRANSFER MATRICES, THE "STAR-TRIANGLE RELATION" OR YANG-BAXTER EQUATION.

The Yang-Baxter Equation

DENOTE

$$\Delta(a,b,c) = \frac{a^2+b^2-c^2}{2ab}.$$

LET US MAKE A ROW OF N VERTICES USING THE BOLTZMANN WEIGHTS a, b, c. FOR SIMPLICITY USE TOROIDAL BOUNDARY CONDITIONS, I.E. IDENTIFY THE LEFT AND RIGHT BOUNDARY EDGES.



USE a, b, c BOLTZMANN WEIGHT AT VERTEX v.

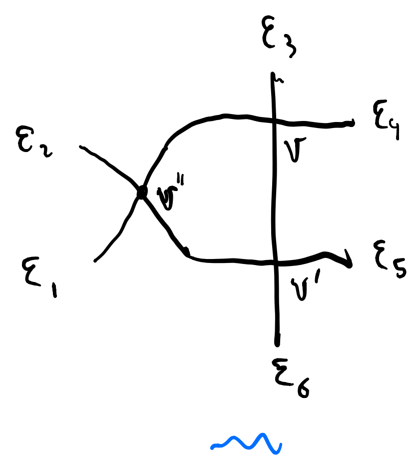
LET $T_{N,a,b,c}$ BE THE CORRESPONDING ROW TRANSFER MATRIX.

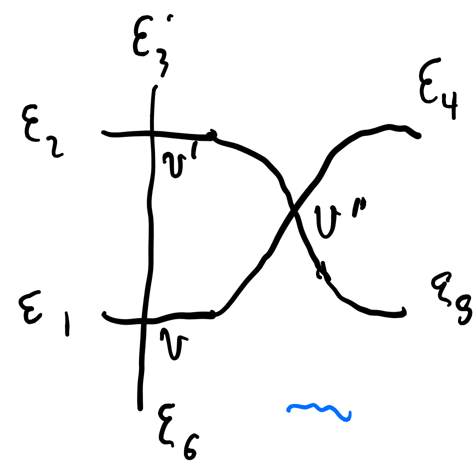
THEOREM (BAXTER) SUPPOSE

$$\Delta(a,b,c) = \Delta(a',b',c').$$

THEN $T_{N,a,b,c}$ AND $T_{N,a',b',c'}$ COMMUTE.

TO PROVE THE COMMUTATIVITY OF THE ROW TRANSFER MATRICES, LET v, v' BE VERTICES WITH BOLTZMANN WEIGHTS a, b, c AND a', b', c', RESPECTIVELY. ASSUMING THAT $\Delta(a,b,c) = \Delta(a',b',c')$ BAXTER PROVES THE EXISTENCE OF a'', b'', c'' SUCH THAT IF v'' IS THE VERTEX WITH BOLTZMANN WEIGHTS a'', b'', c'' THEN FOR ANY SIX SPINS $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6$ THE FOLLOWING TWO MINIATURE SYSTEMS ARE EQUIVALENT:

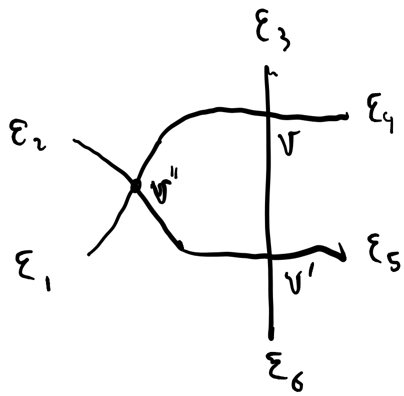


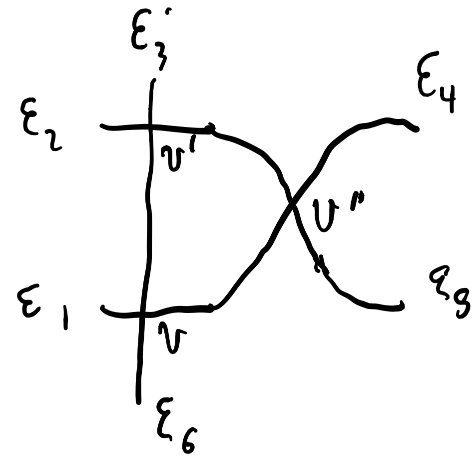


THE VERTEX v'' IS CALLED THE R-MATRIX.

WE ASSUMED $\Delta(a,b,c) = \Delta(a',b',c')$.

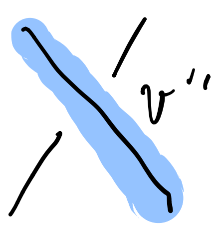
THIS ALSO EQUALS $\Delta(a'',b'',c'')$.





THE SIX BOUNDARY EDGE SPINS ARE FIXED. THE THREE INTERIOR SPINS ARE SUMMED.

THE R-MATRIX IS ROTATED. SO TO DISTINGUISH TWO POSSIBLE ORIENTATIONS WE DRAW IT THUS:



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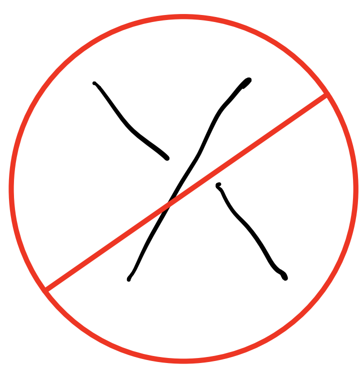
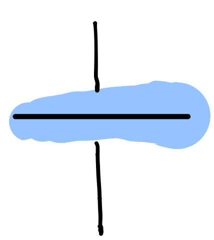
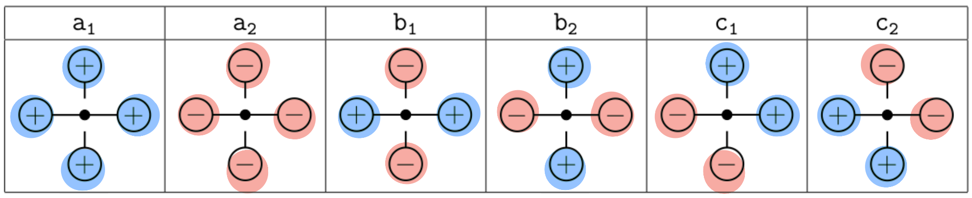


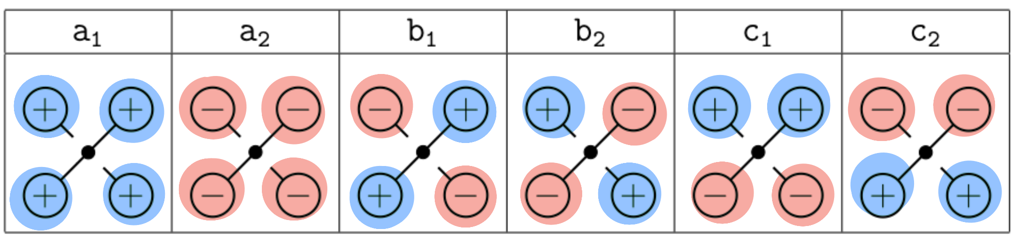
SIMILARLY WE SPECIFY THE ORIENTATION OF v, v':



WITH THESE CONVENTIONS HERE ARE THE LABELLINGS OF THE BOLTZMANN WEIGHTS:

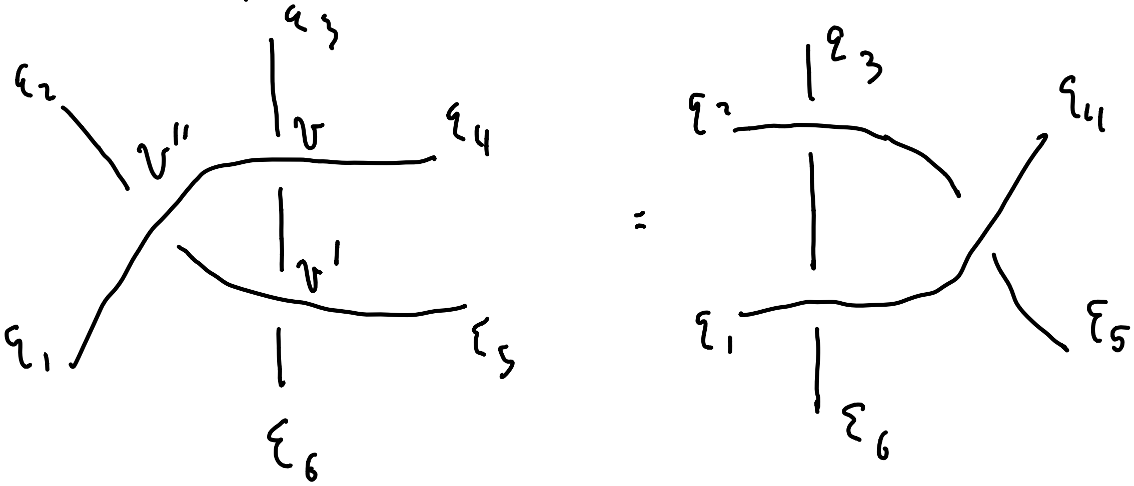


$\underbrace{a_1 \quad a_2}_{a \text{ or } a'}$ $\underbrace{b_1 \quad b_2}_{b \text{ or } b'}$ $\underbrace{c_1 \quad c_2}_{c \text{ or } c'}$

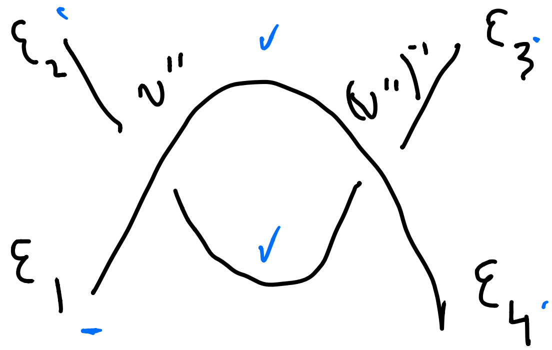


$\underbrace{a_1 \quad a_2}_{a''}$ $\underbrace{b_1 \quad b_2}_{b''}$ $\underbrace{c_1 \quad c_2}_{c''}$

AND THE YANG-BAXTER EQUATION:



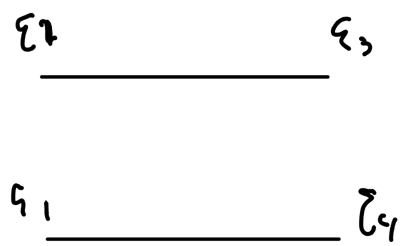
THE R-MATRIX v'' IS INVERTIBLE MEANING THERE IS ANOTHER SET OF BOLTZMANN WEIGHTS TO BE DENOTE $(v'')^{-1}$ SUCH THAT:



$$= \begin{cases} 1 & \text{IF } \varepsilon_1 = \varepsilon_4,\ \varepsilon_2 = \varepsilon_3 \\ 0 & \text{OTHERWISE} \end{cases}$$

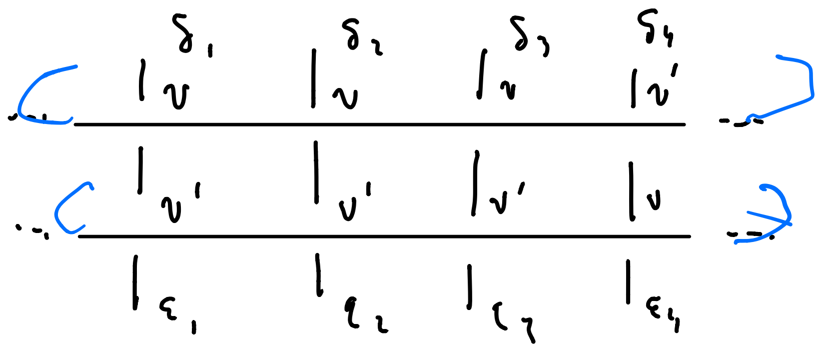
$=$

$=$



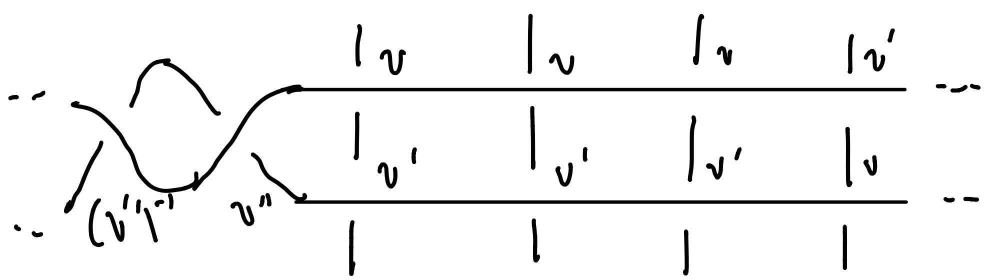
Commutativity of Row Transfer Matrices

$\langle \varepsilon | T_{v'} T_v | \delta \rangle =$

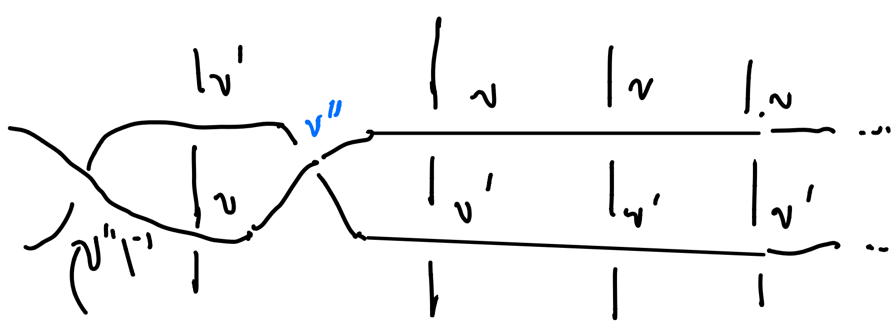


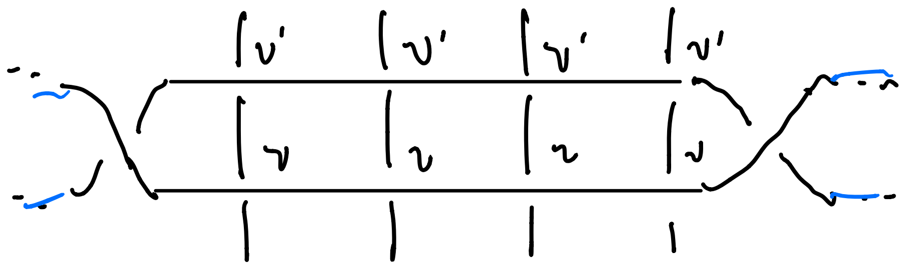
(IDENTIFY LEFT & RIGHT BDY)

$=$



$=$





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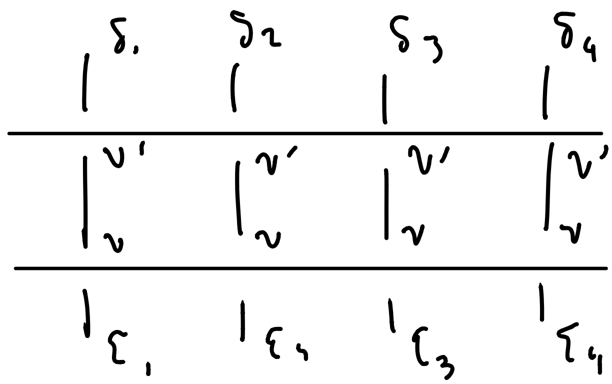
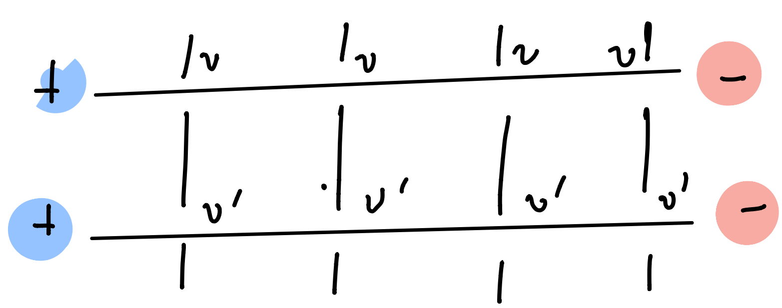


$$= \langle \varepsilon | T_{\nu'} T_{\nu} | \delta \rangle$$

PROVING THE ROW TRANSFER MATRICES COMMUTE !

FOR NON-TOROIDAL BOUNDARY CONDITIONS:



Domain Wall Boundary Conditions

A SIMILAR IDEA WORKS BUT PRODUCES

$$T_{v'} T_v = (*)\, T_v T_{v'}$$

WHERE $(*)$ IS A CONSTANT. INDEED (DROPPING THE FIELD-FREE CONDITION $a_1'' = a_2''$) THE CONSTANT IS a_2''/a_1''.

SUCH "DOMAIN WALL" BOUNDARY CONDITIONS WERE FIRST CONSIDERED BY KOREPIN AND IZERGIN. THEY ARE USED THROUGHOUT THE REST OF THE TALK.

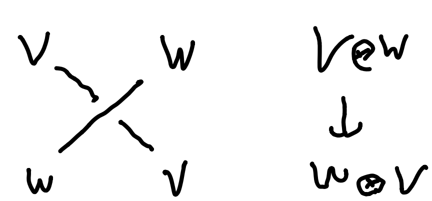
Quantum Groups

SOLUTIONS TO THE YANG-BAXTER EQUATION SUCH AS BAXTER'S EXAMPLE ABOVE AND HIS DEEPER SOLUTION TO THE FIELD-FREE 8-VERTEX MODEL COME FROM QUANTUM GROUPS (DRINFELD, JIMBO). QUANTUM GROUPS ARE HOPF ALGEBRAS THAT CAN BE UNDERSTOOD AS DEFORMATIONS OF LIE GROUPS OR THEIR ENVELOPING ALGEBRA. IF V, W ARE MODULES OF A QUANTUM GROUP H THERE IS AN ISOMORPHISM OF MODULES:

$$\tau R : V \otimes W \longrightarrow W \otimes V$$

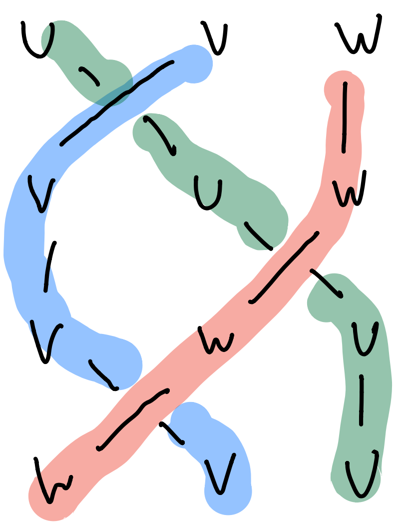
$$R \in H \otimes H, \quad \tau(x \otimes y) = y \otimes x$$

THEN τR IS THE R-MATRIX. IT GIVES A BRAIDING IN THE MODULE CATEGORY. WE VISUALIZE IT THUS:



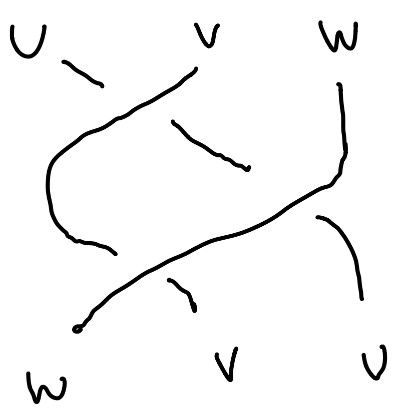
NOW IF U, V, W ARE MODULES WE HAVE AN ISOMORPHISM

$$U \otimes V \otimes W \longrightarrow W \otimes V \otimes U$$

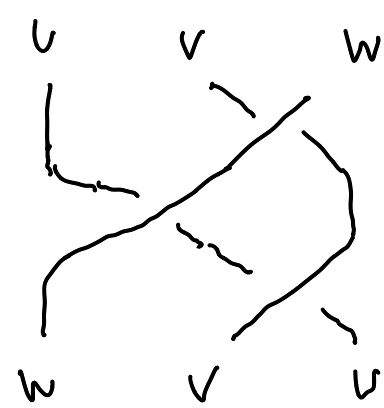


$$\begin{tikzcd} U \otimes V \otimes W \arrow[d, "\tau R \otimes I_W"] \\ V \otimes U \otimes W \arrow[d, "I_V \otimes \tau R"] \\ V \otimes W \otimes U \arrow[d, "\tau R \otimes I_U"] \\ W \otimes V \otimes U \end{tikzcd}$$

OR MORE SIMPLY:



A SECOND ISOMORPHISM:



THESE ISOMORPHISMS ARE EQUAL. (YBE)

ANY 3 MODULES OF A QG GIVES
A SOLUTION TO YBE.

IF $\Delta = \dfrac{a^2+b^2-c^2}{2ab} = \frac{1}{2}(q+q^{-1})$

THE QG $U_q(\widehat{sl_2})$ GIVES RISE TO
BAXTER'S SOLUTION.

$\widehat{sl_2}$: AFFINE sl_2

$U_q(\widehat{sl_2})$: QUANTIZED ENVELOPING ALGEBRA.

$$0 \to \mathbb{C} \longrightarrow \widehat{sl_2} \longrightarrow sl_2 \otimes \mathbb{C}[t,t^{-1}] \to 0$$

CENTRAL EXTENSION

FOR EVERY sl_2-MODULE V, $\widehat{sl_2}$
HAS A FAMILY V_z $(z \in \mathbb{C}^*)$ OF
MODULES. TAKING $V = \mathbb{C}^2$ (STANDARD MODULE)
GIVES BAXTER'S SOLUTIONS.
PARAMETRIZED YBE.

Free-Fermionic YBE

$$a_1 a_2 + b_1 b_2 = c_1 c_2$$

THERE IS A PARAMETRIZED YBE
WITH NON-ABELIAN PARAMETER GROUP

$$GL(2,\mathbb{C}) \times GL(1,\mathbb{C})$$

(KOREPIN: REDISCOVERED BY BRUBAKER, BUMP
FRIEDBERG.)

THIS NONABELIAN PARAMETRIZED YBE
CONTAINS BAXTER'S $U_q(\widehat{sl}_2)$ $q = \sqrt{-1}$.
IT ALSO CONTAINS $U_q(\widehat{gl}(1|1))$
FOR ANY q.

$gl(m|n)$: LIE SUPERALGEBRA

$\widehat{gl}(m|n)$: AFFINIZATION

Tokuyama Ice

LET $z_1, \cdots, z_r$ BE SPECTRAL PARAMETERS,
λ A PARTITION,

$$\Delta_\lambda(z) = \Delta_\lambda(z_1, \cdots, z_r)$$

THE SCHUR FUNCTION (CHARACTER OF $GL_r(\mathbb{C})$).

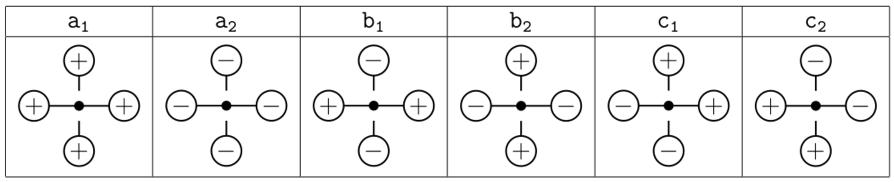
WE DESCRIBE A SYSTEM (TOKUYAMA, HAMEL-KING, BRUBAKER-BUMP-FRIEDBERG) WHOSE PARTITION FUNCTION IS:

$$\left(\prod_{i<j} (1 - v z_j / z_i) \right) \Delta_\lambda(z) =$$

$$\prod_{\alpha \in \Delta^+} (1 - v z^{-\alpha}) \Delta_\lambda(z)$$

$\uparrow$
GL_r ROOT SYSTEM

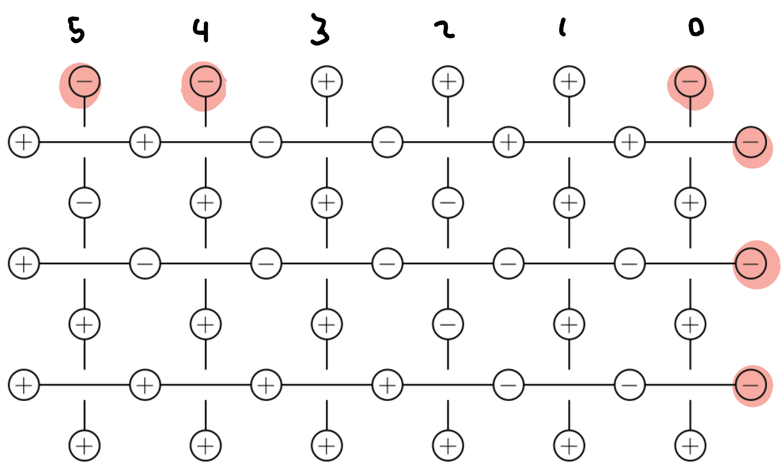
(TOKUYAMA DID NOT CONSIDER ICE MODEL BUT A SUM OVER GTP.)



$1 \qquad z_i \qquad -v \qquad z_i \qquad z_i(1-v) \qquad 1$

IN i-TH ROW USE THESE BOLTZMANN WEIGHTS (FREE-FERMIONIC: $U_q(\widehat{\mathfrak{gl}}(1|1))$)

THE PARTITION λ IS ENCODED IN BOUNDARY CONDITIONS.



$\lambda = (3,3,0) \qquad \lambda+\rho = (5,4,0)$

$\rho = (2,1,0)$ "WEYL VECTOR"

Whittaker Functions

LET F BE A LOCAL FIELD LIKE $\mathbb{Q}_p$ OR $\mathbb{R}$.
REPRESENTATIONS OF $GL(r, F)$ ARE USUALLY INFINITE-DIMENSIONAL. IF (π, V) IS A IRR. REP'N THEN THERE EXISTS AT MOST ONE LINEAR FUNCTIONAL $\omega : V \longrightarrow \mathbb{C}$ SUCH THAT

$$\omega\left(\pi\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ x_{ij} & & 1 \end{pmatrix} v\right) = \qquad \begin{pmatrix} 1 & & \\ * & \ddots & \\ & & 1 \end{pmatrix} = N_-(F)$$

$$\psi\left(x_{21} + x_{32} + \dots + x_{r,r-1}\right)\omega(v)$$

$\uparrow$

ADDITIVE CHARACTER OF F.

IF $v \in V$ $\quad W_v(g) = \omega(\pi(g)v)$ IS A **WHITTAKER FUNCTION.**

FOR $GL(2, \mathbb{R})$ THESE ARE CONFLUENT HYPERGEOMETRIC FUNCTIONS (E.T. WHITTAKER)

Casselman-Shalika

LET F BE NONARCHIMEDEAN. THERE IS A FAMILY $M(z)$ OF PRINCIPAL SERIES REPS $I(z)$ OF $GL(r,F)$ PARAMETRIZED BY SATAKE-LANGLANDS PARAMETERS

$$z=(z_1,\cdots,z_r)\in(\mathbb{C}^\times)^r.$$

LET $K=GL(r,\mathcal{O})$ BE THE MAX'L COMPACT SUBGROUP.

$\mathcal{O}$: INTEGERS IN F

$I(z)$ HAS A UNIQUE K-FIXED VECTOR AND THE CORRESPONDING WHITTAKER FUNCTION SATISFIES:

$$W\begin{pmatrix}\varpi^{\lambda_1} & & \\ & \ddots & \\ & & \varpi^{\lambda_r}\end{pmatrix}=\prod_{\alpha\in\Delta^+}(1-vz^{-\alpha})\,\Delta_\lambda(z)$$

ϖ: PRIME ELEMENT

WHICH WE RECOGNIZE FROM TOKUYAMA MODEL.

Colored Models

TO MAKE A DEEPER CONNECTION WITH THE REPRESENTATION THEORY IT IS NECESSARY TO REALIZE DEEPER LEVEL IWAHORI WHITTAKER FUNCTIONS FIXED BY THE SMALLER IWAHORI SUBGROUP. THERE ARE $|W| = r!$ INDEPENDENT IWAHORI WHITTAKER FUNCTIONS, RELATED BY THE ACTION OF THE AFFINE HECKE ALGEBRA THROUGH DEMAZURE WHITTAKER OPERATORS.

A KEY IDEA BEHIND THESE MODELS WAS SEEN IN BOSONIC MODELS OF BORODIN-WHEELER (2018). THIS IS THE USE OF COLORED MODELS.

BORODIN-WHEELER (2018)	BRUBAKER-BUMP-BUCIUMAS-GUSTAFSON (2019)
BOSONIC	FERMIONIC
$U_q(\widehat{\mathfrak{sl}}_{r+1})$	$U_q(\widehat{\mathfrak{gl}}(r\|1))$.

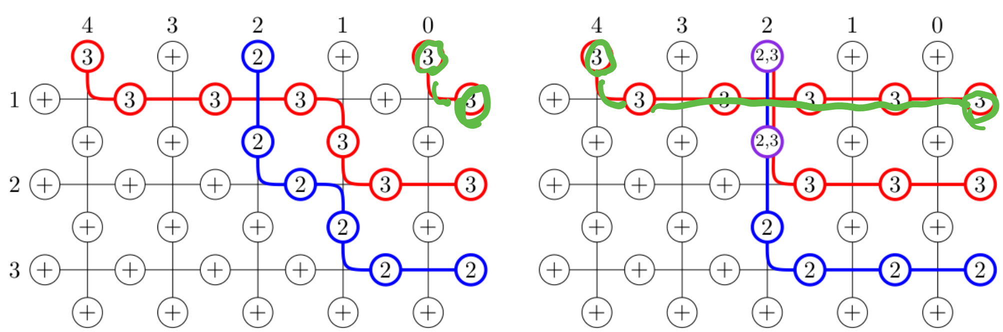
IN THE COLORED MODELS THE HORIZONTAL EDGES CORRESPOND TO $r+1$ DIMENSIONAL STANDARD MODULES OF $U_q(\widehat{\mathfrak{gl}}(r|1))$.

EACH EDGE CARRIES ONE OF THE FOLLOWING SPINS:

$+$ $\qquad\qquad$ $\underbrace{c_1, c_2, \ldots}_{r \text{ colors}}$

VERTICAL EDGES MAY CARRY MORE THAN ONE COLOR BUT (FERMIONIC) EACH COLOR WITH MULTIPLICITY ≤ 1.

WE MAY DRAW COLORED LINES CONNECTING THE BOUNDARIES:



(PARAHORIC)

Iwahori Whittaker Functions

THE IWAHORI SUBGROUP J IS SMALLER THAN THE MAXIMAL COMPACT $K = GL(r, \mathcal{O})$, SO IT HAS MORE FIXED VECTOR.

$$J = \{ k \in K \mid k \text{ lower triangular} \bmod \mathfrak{p} = (\varpi) \}.$$

SPACE OF IWAHORI WHITTAKER FUNCTIONS SATISFY DEMAZURE-WHITTAKER RECURSIONS

CORRESPOND TO YBE.

$$\mathfrak{T}_i f(\mathbf{z}) = \frac{f(\mathbf{z}) - f(s_i\mathbf{z})}{\mathbf{z}^{\alpha_i} - 1} - v\frac{f(\mathbf{z}) - \mathbf{z}^{-\alpha_i} f(s_i\mathbf{z})}{\mathbf{z}^{\alpha_i} - 1}.$$

$$T_i^2 = (v-1)T_i + v$$

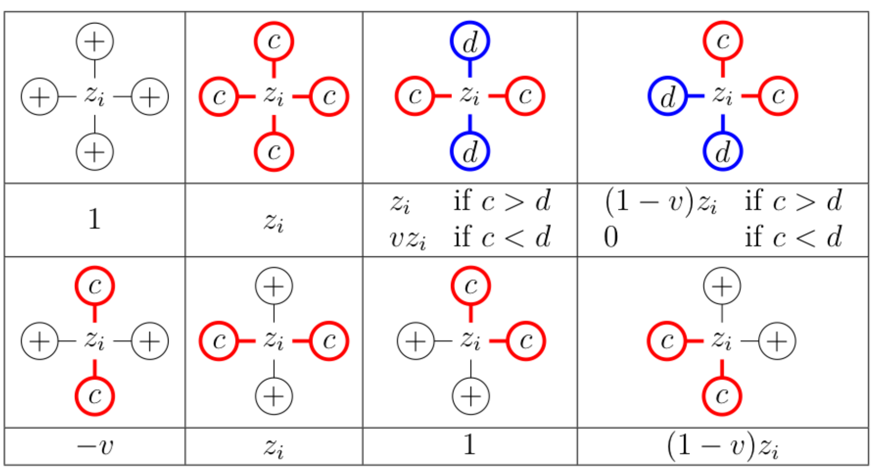
(v^{-1} = RESIDUE FIELD CARDINALITY)

$$\phi_{s_i w}(g) = \begin{cases} T_i \phi_w(g) & s_i w > w \\ T_i^{-1} \phi_w(g) & s_i w < w \end{cases}$$

THEOREM: CHOOSING g FROM A SET OF COSET REPS FOR $N_-(F)\backslash GL(r,F)/J$ AND ϕ_w FROM A BASIS OF IWAHORI WHITTAKER FUNCTIONS THERE ARE COLORED SYSTEMS WHOSE PARTITION FUNCTIONS SATISFY

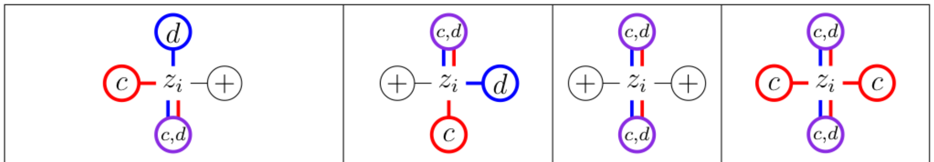
$$Z(\mathfrak{S}_{g,w}) = \phi_w(g)$$

DEMAZURE RECURSIONS CAN BE PROVED USING A YANG-BAXTER EQUATION.



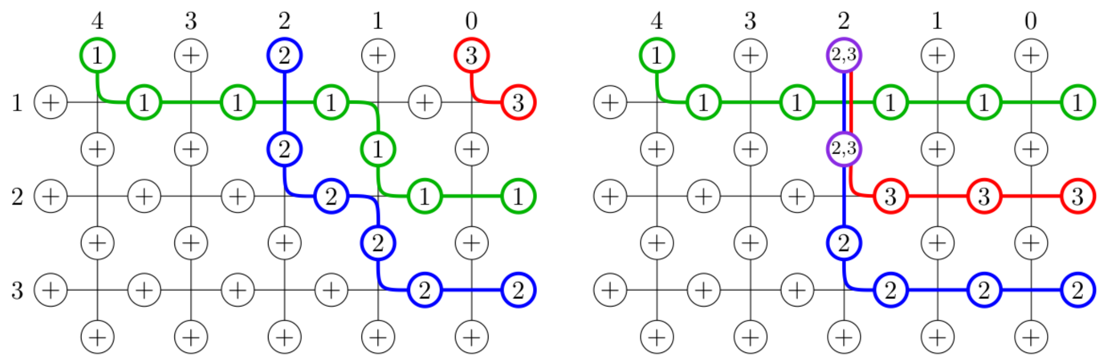
SOME OF THE BOLTZMANN WEIGHTS (THOSE FOR WHICH NO EDGE CARRIES 2 COLORS)

VERTICAL EDGES CAN CARRY 2 OR MORE COLORS. HERE ARE SOME MORE BOLTZMANN WEIGHTS.



$(1-v)z_i$ if $c>d$, $(-v)(1-v)z_i$ if $c<d$.	v if $c>d$, $-v$ if $c<d$.	$(-v)^2$	z_i if $c>d$, vz_i if $c<d$.

HERE ARE TWO SAMPLE STATES:



THESE DATA ENCODE g.

ENCODING DATA:

$+$ $+$ $\vdots$ $+$

$$Z(\mathfrak{S}) = \phi_\omega(g)$$

$+$ $\cdots$ $+$

THESE DATA ENCODE ω

Supersymmetric Models

LATTICE MODELS RELATED TO

$U_q(\widehat{\mathfrak{gl}}(r|n))$ WERE FOUND INDEPENDENTLY

BY BRUBAKER, BUMP, BUCIUMAS AND GUSTAFSSON

AND BY AGGARWAL, BORODIN AND WHEELER.

BOTH PAPERS APPEARED VERY RECENTLY

IN ARXIV:

arXiv: 2012.15778 (BBBG)
arXiv: 2101.01605 (ABW)

BOTH FERMIONIC!

WE WILL REPORT ON THE BBBG PAPER.

THIS CONCERNS IWAHORI WHITTAKER

FUNCTIONS ON CENTRAL EXTENSIONS:

$$1 \to \mu_n \to \widetilde{GL}_r^{(n)}(F) \to GL_r(F) \to 1$$

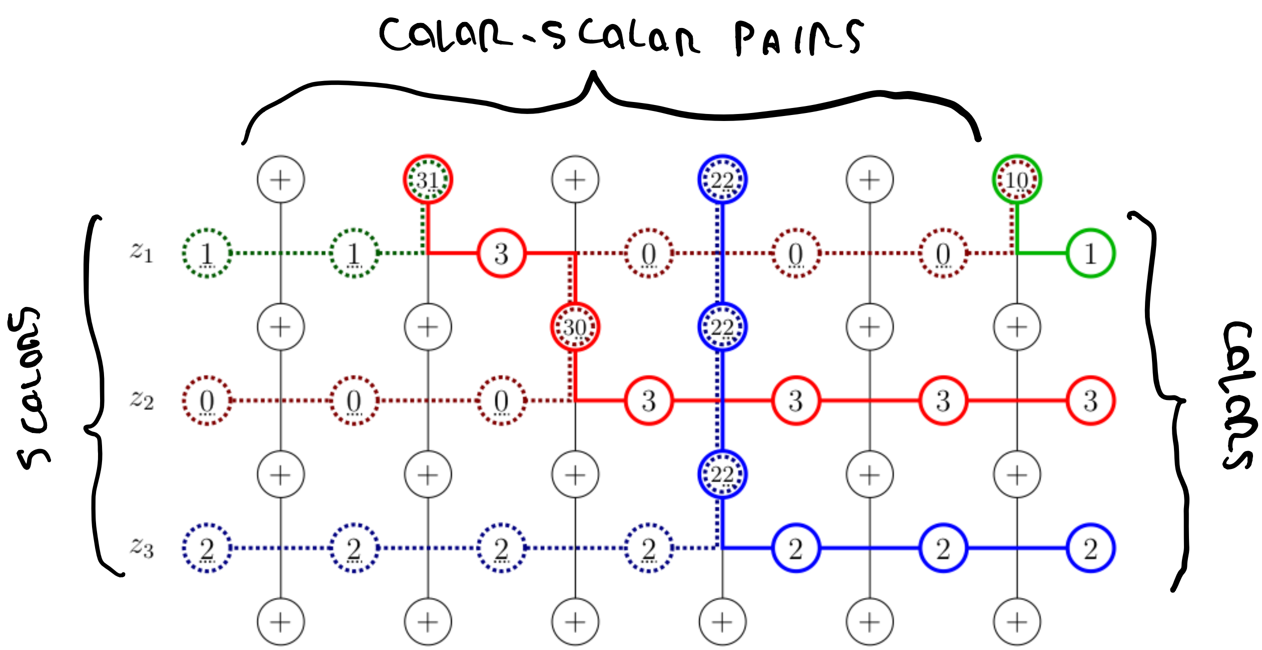
F IS A NONARCHIMEDEAN LOCAL FIELD CONTAINING

2n DISTINCT 2n-TH ROOTS OF UNITY.

IN THIS SITUATION WE REQUIRE r DISTINCT COLORS AND n DISTINCT SUPERCOLORS (SCOLORS).

HORIZONTAL EDGES MUST CARRY EXACTLY ONE COLOR OR ONE SCOLOR (NEVER BOTH).

A VERTICAL EDGE CAN CARRY ONE OR MORE (OR LESS!) COLOR-SCOLOR PAIR.

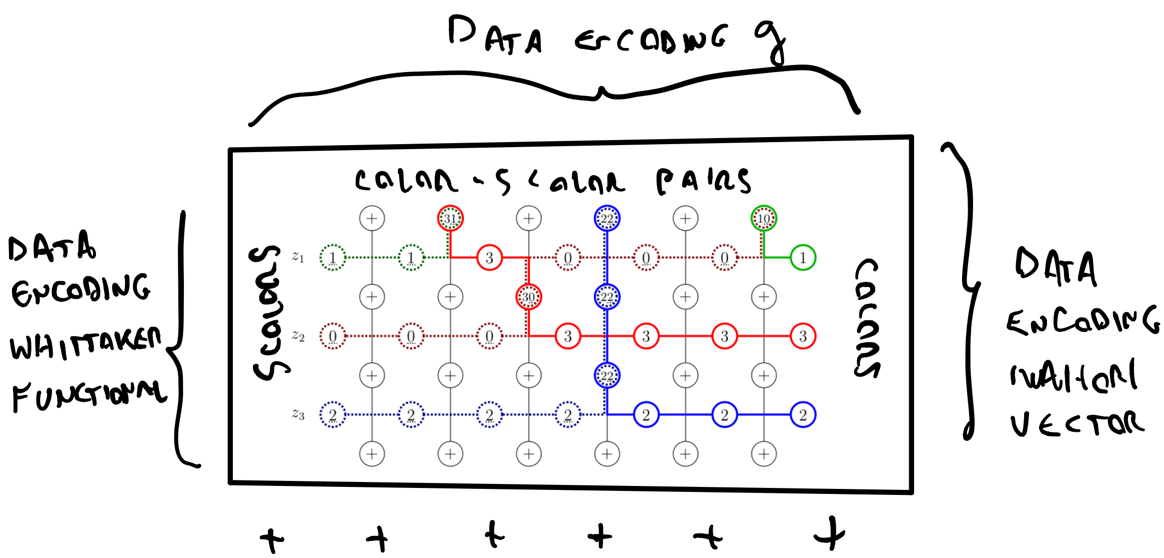


COLORED LINES MOVE DOWN AND TO THE RIGHT

SCOLORED LINES MOVE DOWN AND TO THE LEFT.

PRINCIPAL SERIES REPRESENTATIONS OF $\widetilde{GL}_r^{(n)}(F)$ HAVE $|W| = r!$ IWAHORI FIXED VECTORS AND n^r WHITTAKER FUNCTIONALS.

WE ENCODE THESE DATA IN THE BOUNDARY CONDITIONS:



NOW THERE ARE STILL REPRESENTATIONS OF THE AFFINE HECKE ALGEBRA BY (VECTOR) DEMAZURE-WHITTAKER OPERATORS BUT THESE ARE NOW COMPLEX.

THIS INTRICATE SETUP IS EXACTLY MIRRORED IN THE LATTICE MODELS.